

# Neutrino Physics: Open Theoretical Questions

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- What we have learned?
- Open theoretical questions
- Bottom-up
- How we might go...

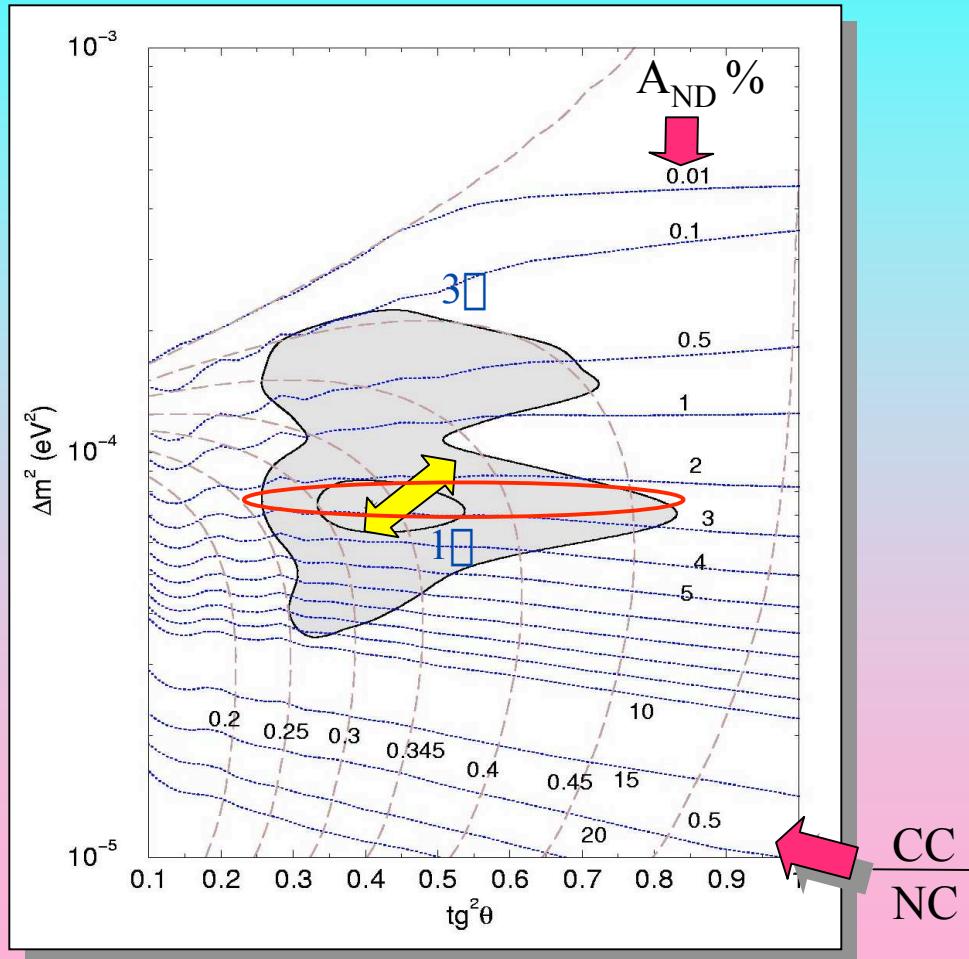
# **1. What we have learned?**

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Neutrino masses and lepton mixing:  
Summary

# Solar Neutrinos

P.de Holanda, A.S.



Lines of constant CC/NC ratio and  
Day-Night asymmetry at SNO

Best fit point:

$$\begin{aligned} \Delta m_{12}^2 &= 7 \cdot 10^{-5} \text{ eV}^2 \\ \tan^2 \theta_{12} &= 0.4 \\ \sin^2 \theta_{13} &\sim 0 \end{aligned}$$

LM MSW

Any problem?

- ~ 2 higher Ar-production rate than Homestake result
- Absence of the upturn of the spectrum

# Survival probability

Light sterile neutrino

$$R_{\square} = \square m_{01}^2 / \square m_{21}^2$$

$\square$  - mixing angle of sterile neutrino

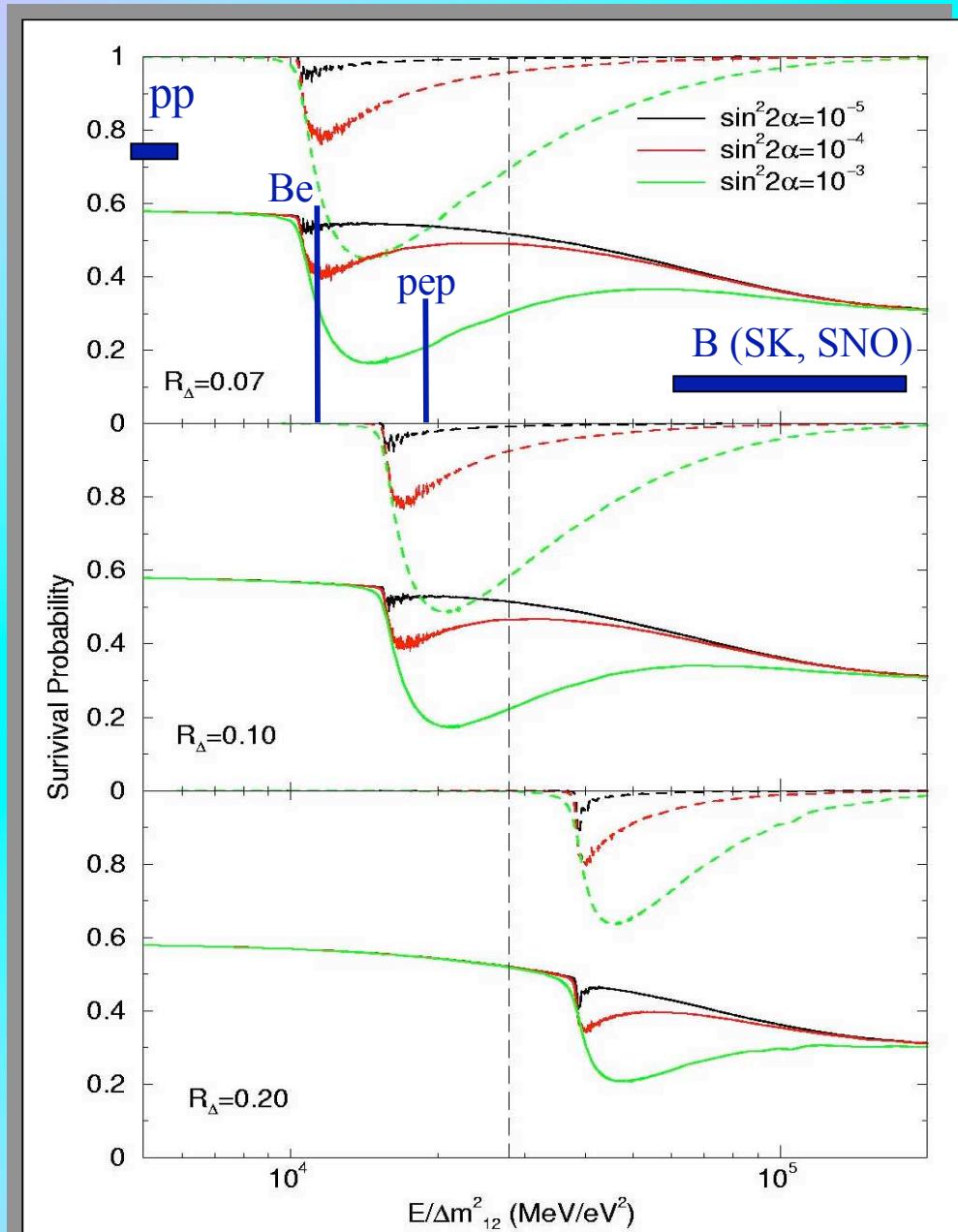
Dip in the survival probability:

- reduces the Ar-production rate
- suppresses the upturn of spectrum

Motivation for the low energy solar neutrino experiments

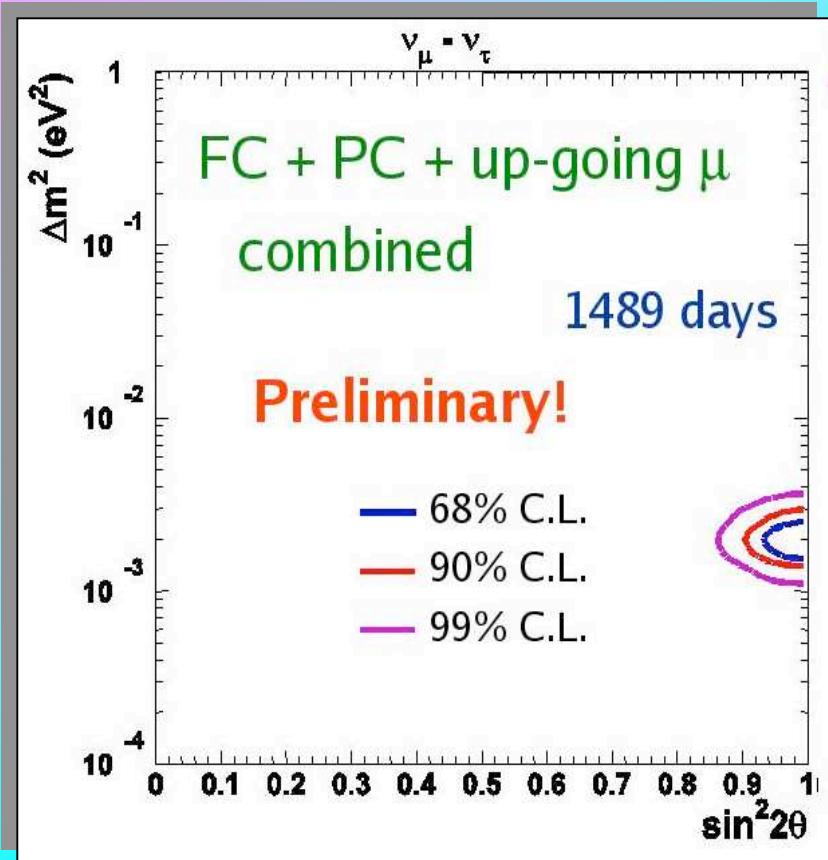
BOREXINO, KamLAND

MOON, LENS ...



# Atmospheric Neutrinos

SuperKamiokande:



Best fit point:

$$\sin^2 2\theta_{23} = 1.0$$

$$\Delta m_{32}^2 = 2.0 \cdot 10^{-3} \text{ eV}^2$$

$$\Delta m_{32}^2 = (1.3 - 3.0) \cdot 10^{-3} \text{ eV}^2$$
$$\sin^2 2\theta_{23} > 0.9$$

(90 % C.L.)

Confirmed by  
MACRO,  
SOUDAN  
K2K

Combined analysis of CHOOZ,  
atmospheric (SK) and solar data:

$$\sin^2 2\theta_{13} < 0.067 \text{ (3\sigma)}$$

G.L. Fogli et al, hep-ph/p0308055

# LMA oscillations of atmospheric neutrinos

Excess of the e-like events in sub-GeV

$$\frac{F_e}{F_e^0} - 1 = P_2(r c_{23}^2 - 1)$$

``screening factor''

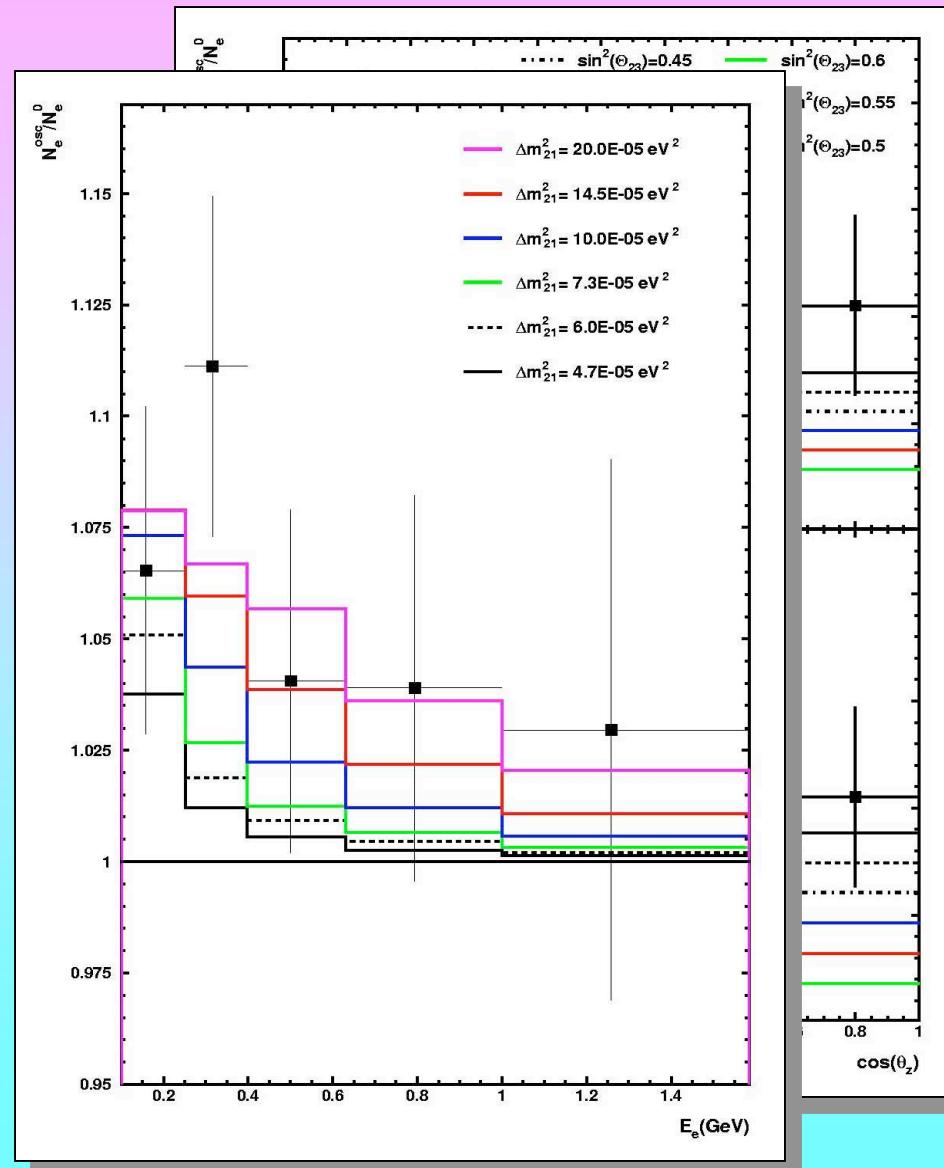
$P_2 = P(\Delta m_{12}^2, \theta_{12})$  is the  $2\bar{\nu}$  transition probability

In the sub-GeV sample  $r = F_\mu^0 / F_e^0 \sim 2$

→ The excess is zero for maximal 23- mixing

Searches of the excess can be used to restrict deviation of the 2-3 mixing from maximal

Zenith angle and energy dependences of the e-like events



# Conversion of neutrinos from SN1987A

- After KamLAND: one must take into account conversion effects of supernova neutrinos

$$F(\bar{\nu}_e) = F^0(\bar{\nu}_e) + p \bar{\nu} F^0$$

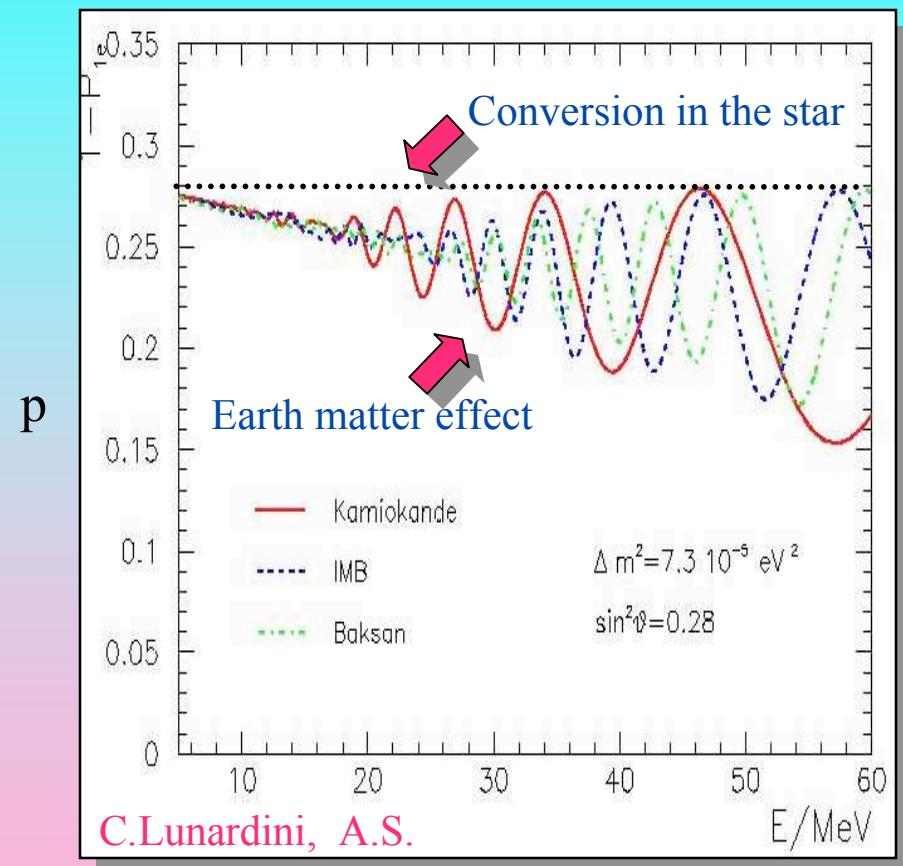
$p$  is the permutation factor

$$\bar{\nu} F^0 = F^0(\bar{\nu}_{\bar{\nu}}) - F^0(\bar{\nu}_e)$$

- $p$  depends on distance traveled by neutrinos inside the earth to a given detector:

$$d = \begin{cases} 4363 \text{ km} & \text{Kamioka} \\ 8535 \text{ km} & \text{IMB} \\ 10449 \text{ km} & \text{Baksan} \end{cases}$$

- The earth matter effect can partially explain the difference of Kamiokande and IMB: spectra of events

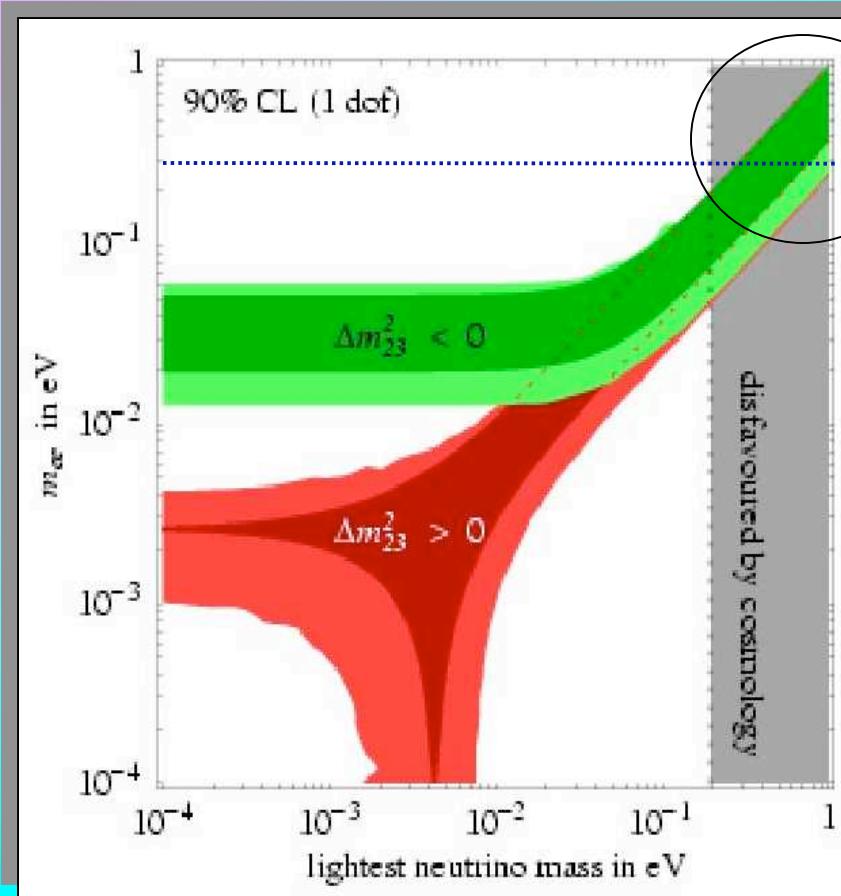


- Normal hierarchy is preferable  
H. Minakata, H. Nunokawa,  
J Bahcall, D Spergel, A.S.

# Absolute scale of mass

F. Feruglio, A. Strumia, F. Vissani

Neutrinoless double beta decay



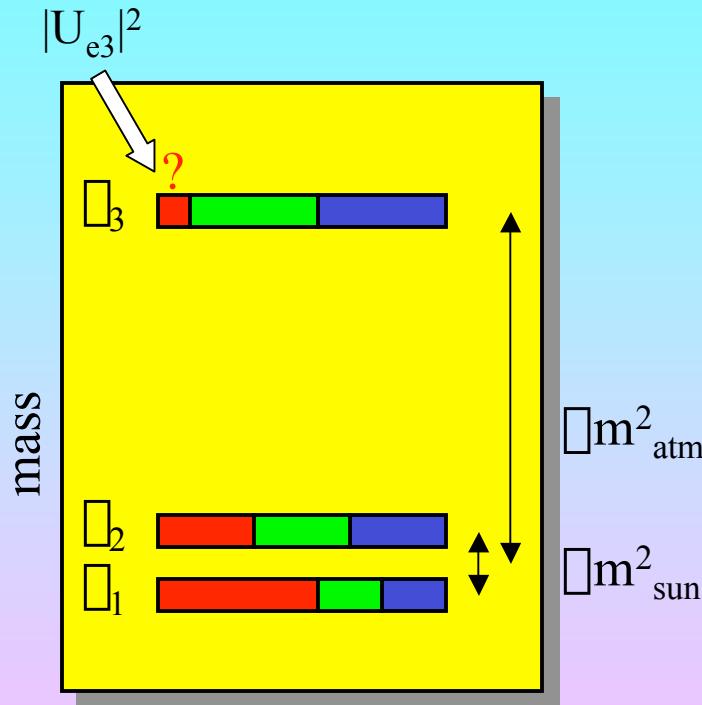
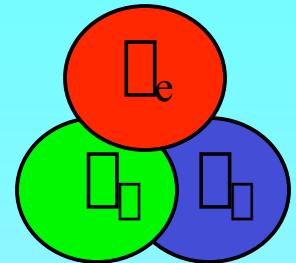
Kinematic searches, cosmology

Sensitivity limit

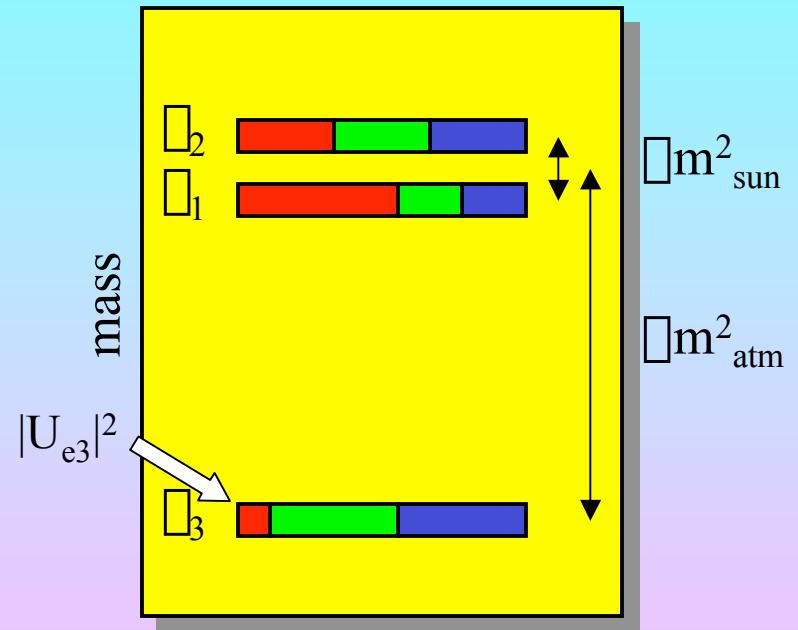
$$m_{ee} = \sum_k U_{ek} m_k e^{i\phi(k)}$$

Both cosmology and double beta decay have similar sensitivities

# Mass spectrum and mixing



Normal mass hierarchy  
(ordering)



Inverted mass hierarchy  
(ordering)

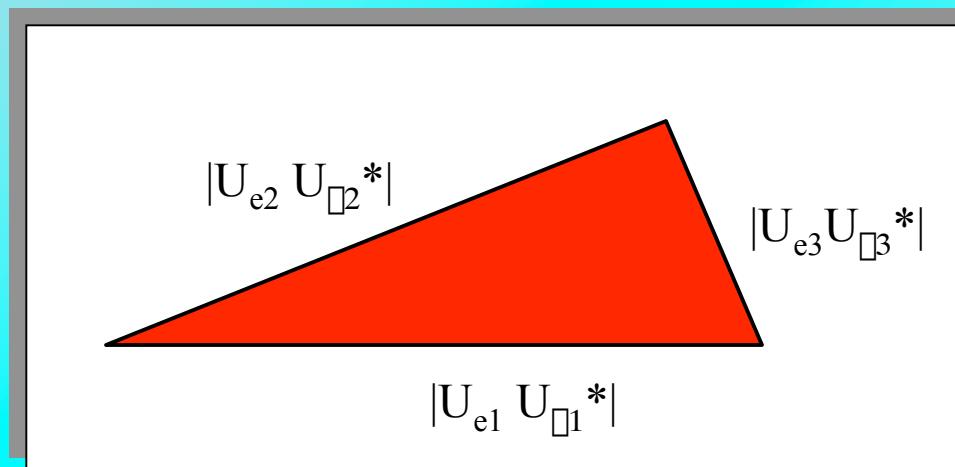
- Type of mass spectrum: with Hierarchy, Ordering, Degeneracy → absolute mass scale
- Type of the mass hierarchy: Normal, Inverted
- $U_{e3} = ?$

# Leptonic Unitarity Triangle

$$U_{PMNS} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} 0.79 - 0.86 & 0.50 - 0.61 & 0.0 - 0.16 \\ 0.24 - 0.52 & 0.44 - 0.69 & 0.63 - 0.79 \\ 0.26 - 0.52 & 0.47 - 0.71 & 0.60 - 0.77 \end{pmatrix}$$

Global fit of the oscillation data [1]

M.C. Gonzalez-Garcia ,  
C. Pena-Garay



- $|U_{e3}| = 0.16$
- nearly best fit values of other angles

Can we reconstruct the triangle?

Can we use it to determine the CP-violating phase? Y. Farsan, A.S.

Problem: coherence (we deal with coherent states and  
not mass eigenstates of neutrinos)

A Yu Smirnov



# Sterile neutrino $(3+1)$ -scheme $(3+2)$ ?

O. Peres, A.S.

M. Sorel, J. Conrad, M. Shaevitz

Ultimate oscillation  
anomaly?

## CPT-violation

After KamLAND:

G. Barenboim,  
L. Borissov, J. Lykken

## Non-standard Interactions

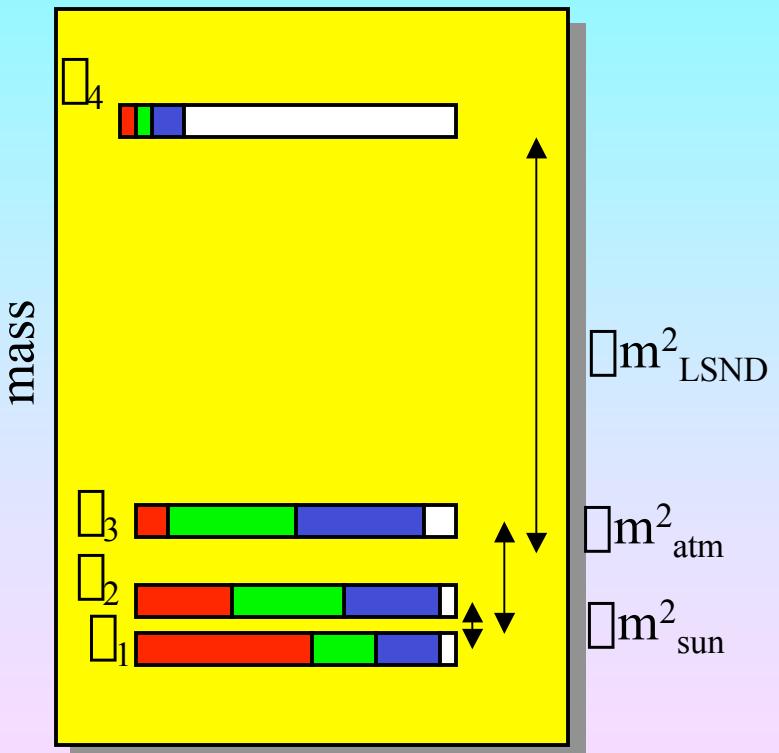
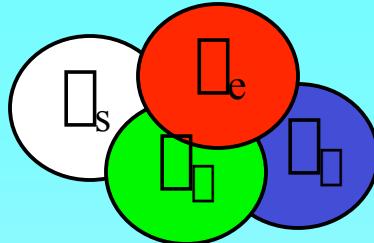
K.Babu, S Pakvasa

Disfavored by a  
new analysis of  
KARMEN  
collaboration

Disfavored by  
atmospheric  
neutrino data,  
no compatibility  
of LSND and  
all-but LSND data

below  $3\sigma$ -level  
M.C. Gonzalez-Garcia,  
M. Maltoni, T. Schwetz

# (3 + 1)



The problem is

$$P \sim |U_{e4}|^2 |U_{\bar{\nu}4}|^2$$

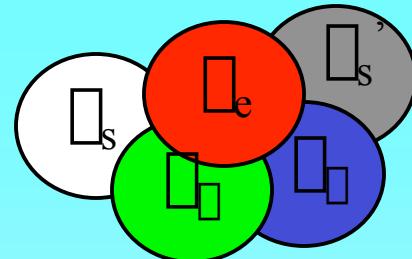
Restricted by short baseline experiments CHOOZ, CDHS, NOMAD  
2 - 3 times below the observed probability

Generic possibility of interest even independently of the LSND result

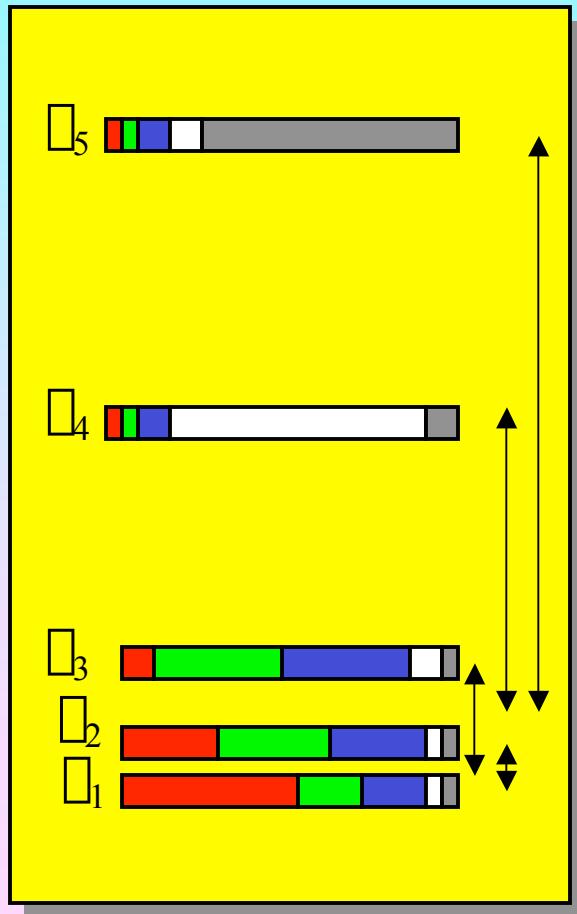
Generation of large mixing of active neutrinos due to small mixing with sterile state

Produces uncertainty in interpretation of results

# 3 + 2 scheme



mass



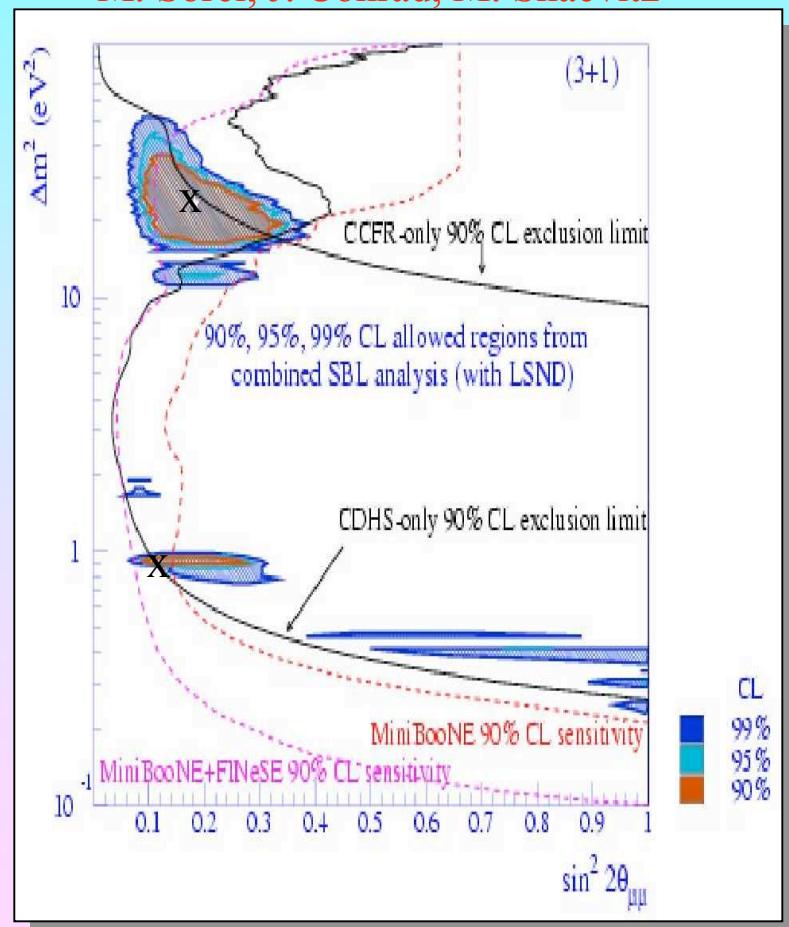
$m^2_{\text{LSND}}$

$m^2_{\text{LSND}}$

$m^2_{\text{atm}}$

$m^2_{\text{sun}}$

M. Sorel, J. Conrad, M. Shaevitz



FINeSE

# Main features

- Smallness of masses:

$$m_\ell < (1 - 2) \text{ eV}$$

$$m_\ell \ll m_l, m_q$$

$$m_\ell > \sqrt{|m_{23}^2|} > 0.04 \text{ eV}$$

(at least for one mass)

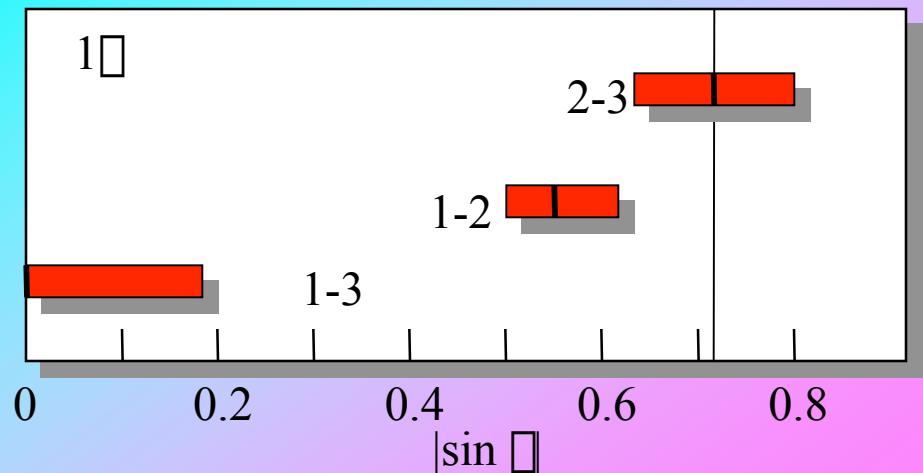
- Hierarchy of mass squared differences:

$$|\Delta m_{12}^2 / \Delta m_{23}^2| = 0.01 - 0.15$$

- No strong hierarchy of masses:

$$|m_2/m_3| > |\Delta m_{12}^2 / \Delta m_{23}^2| = 0.18 \begin{array}{l} + 0.22 \\ - 0.08 \end{array}$$

- Bi-large or large-maximal mixing between neighboring families (1- 2) (2- 3):



- Small mixing between remote families (1- 3):

# ...and unknown

Several key elements  
are unknown yet  
which leads to variety  
of possible interpretations

$$\sin \theta_{13}$$

test of mechanisms  
of the lepton mixing  
enhancement

Type of mass hierarchy  
ordering of states  
(sign of  $m_{13}^2 = m_1^2 - m_3^2$ )

- normal
- inverted

Existence of  
new neutrino  
states

Absolute mass  
scale,  $m_1$   
type of spectrum

- - hierarchical
- partially degenerate
- quasi degenerate

CP-violating  
phases, especially  
Majorana phases

Deviation of  
2-3 and 1-2 mixings  
from maximal

Phenomenological  
and experimental  
problems

## 2. Open theoretical questions

**What does all this mean?**

(results on neutrino  
masses and mixing)

**Old...**

## **What is the origin of neutrino mass?**

- we do not know yet the origin of quark and charged lepton masses where information is more complete
- for neutrinos the problem can be even more complex
- the hope is that neutrinos can shed some light on whole problem of fermion masses

## **Why neutrino masses are small?**

- small in comparison with charged leptons and quarks masses
- what are relations with other mass scales in nature?  
e.g., dark energy scale?

**and New:**

# **How the observed pattern of the lepton mixing is generated?**

- two large mixings and one small (zero)?
- one maximal mixing?
- what are relations between mixing angles?

# **Why the lepton mixing is large? Why it is so different from quark mixing?**

- may be correct question is why the quark mixing is so small?  
In quark sector the smallness of mixing can be related to strong mass hierarchy

# **Do neutrinos show certain flavor or horizontal symmetry?**

- if so, is this symmetry consistent with quark masses and mixing?
- ad hoc introduced symmetries for neutrinos only do not look appealing

# **Are results on neutrino masses and mixing consistent with**

- quark-lepton symmetry?
- Grand Unification?

## **If new light (sterile) neutrino(s) exist**

- what is their nature?
- why they are light?

Experimental and  
phenomenological  
problem



# **What are implications of the neutrino results for**

- GUT**
- SUSY**
- Extra Dimensions?**
- Strings**

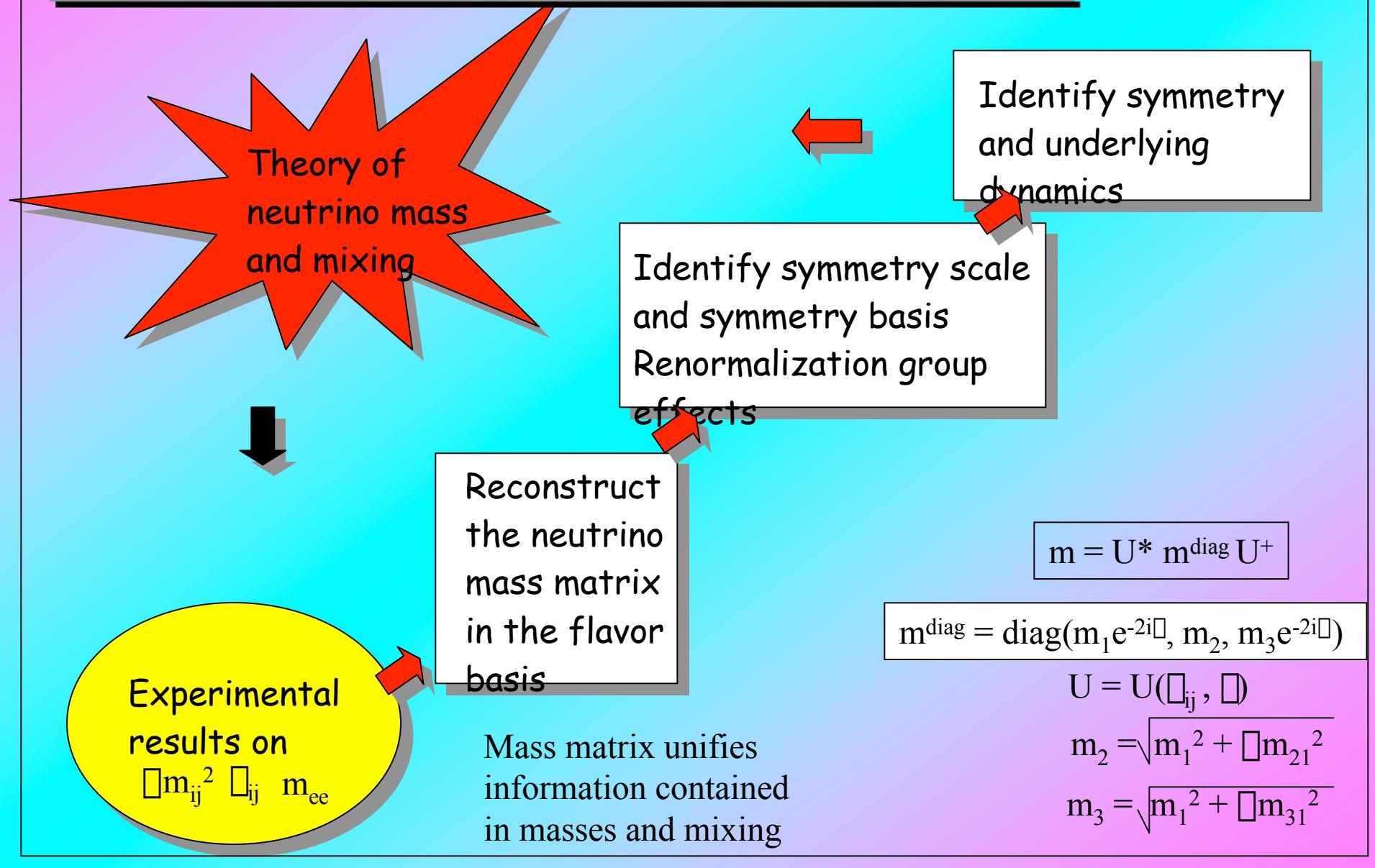
*vice versa*

## **What these theories can tell us about neutrinos?**

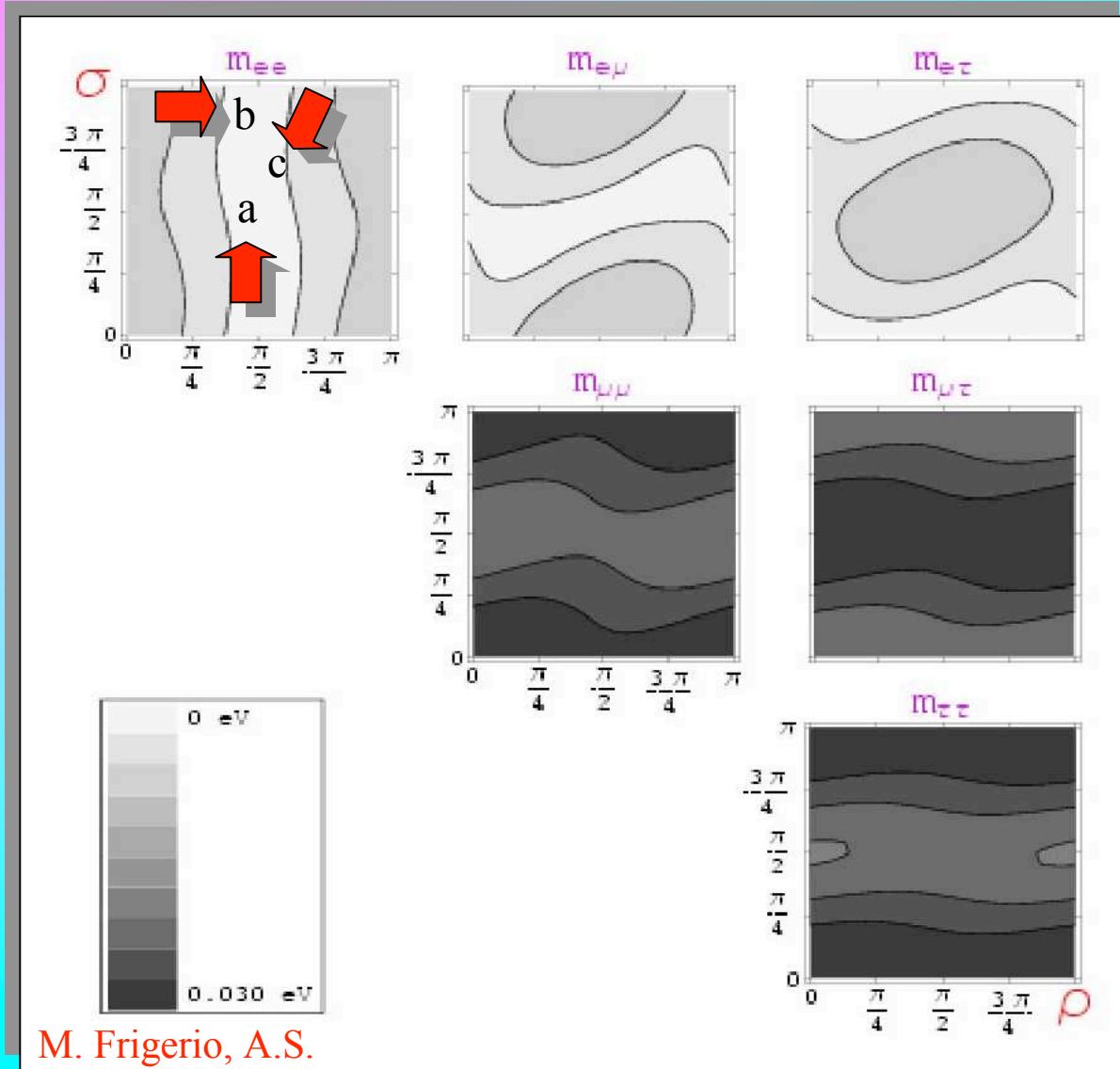
# **3. Bottom-Up**

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# Bottom -Up and Top-Down



# Normal hierarchy



M. Frigerio, A.S.

$$m_3/m_2 = 5$$

$$m_1 = 0.006 \text{ eV}$$

$$\sin^2 2 \theta_{23} = 1$$

$$\sin \theta_{13} = 0.1 \quad \theta = 0$$

a).

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

b).

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

c).

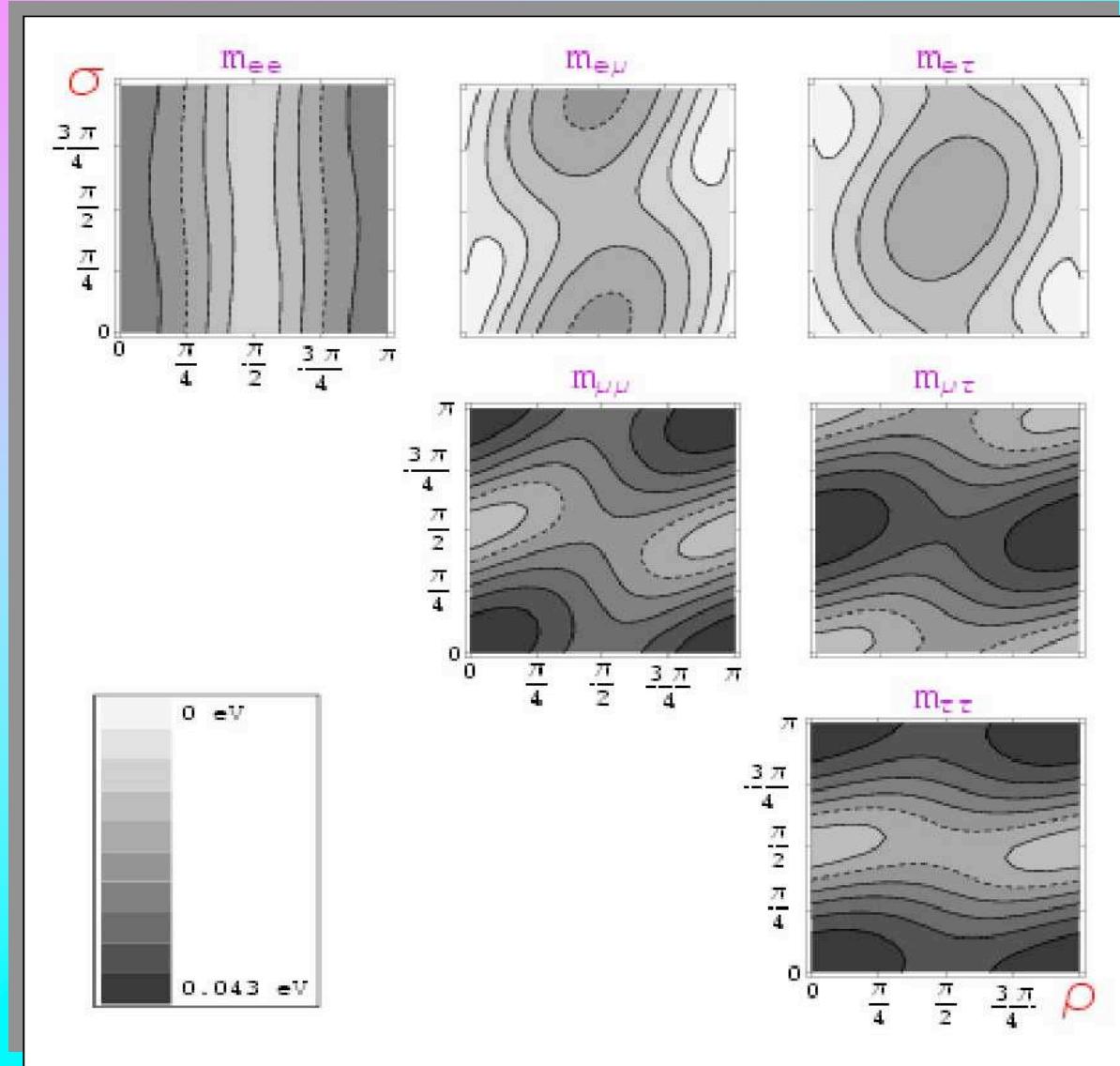
$$\begin{pmatrix} q^2 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

$$q \sim 0.2$$

$$\begin{pmatrix} q^4 & q^3 & q^2 \\ q^3 & q^2 & q \\ q^2 & q & 1 \end{pmatrix}$$

$$q \sim 0.7$$

# Normal ordering



$$m_3/m_2 = 2$$

$$m_1 = 0.027 \text{ eV}$$

$$\sin^2 2\theta_{23} = 1$$

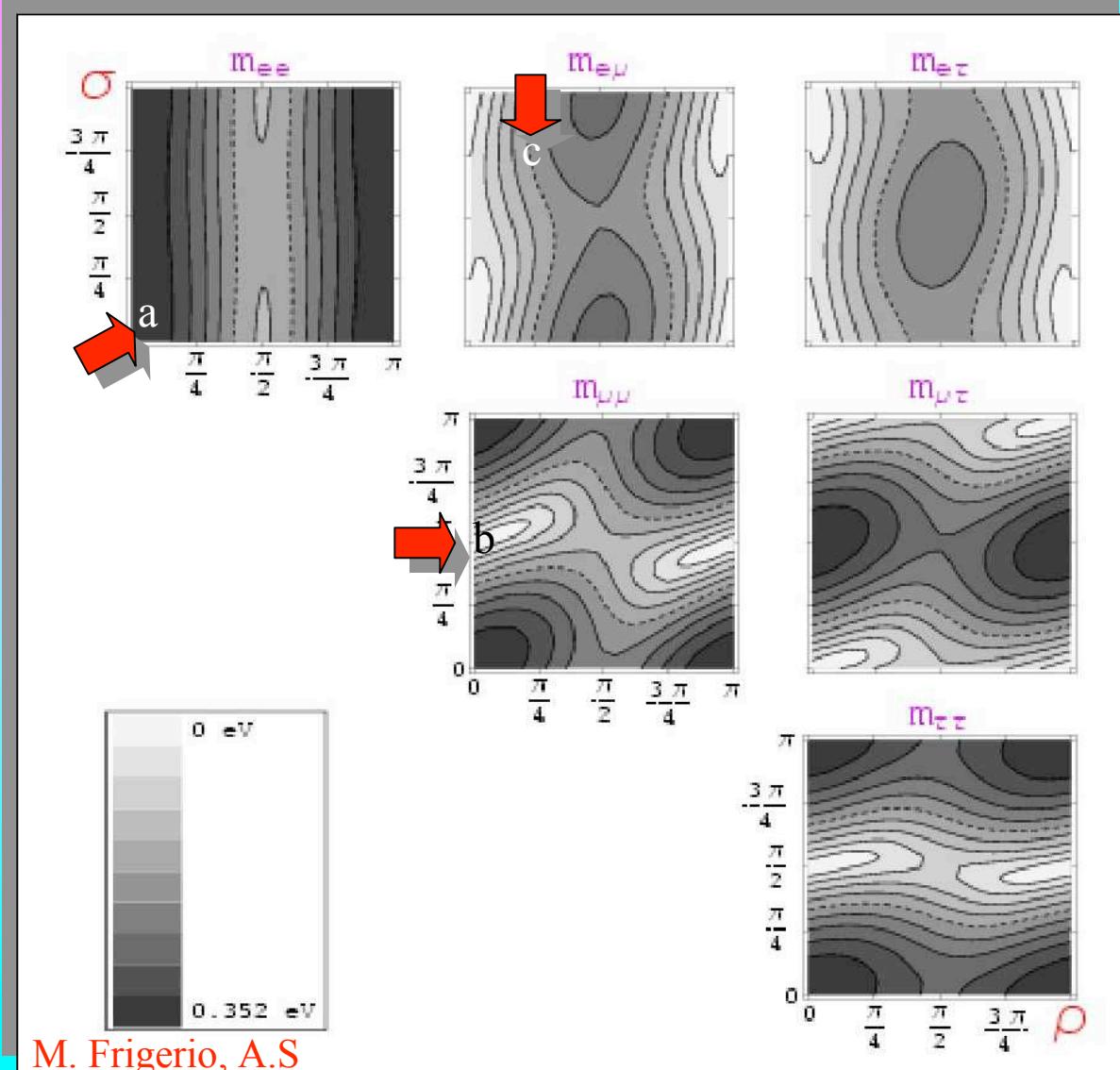
$$\sin \theta_{13} = 0.1 \quad \theta = 0$$

Flavor alignment

$$\begin{pmatrix} q^4 & q^3 & q^2 \\ q^3 & q^2 & q \\ q^2 & q & 1 \end{pmatrix}$$

$$q \sim 0.7$$

# Quasi-degeneracy



$$m_3/m_2 = 1.01$$

$$m_1 = 0.35 \text{ eV}$$

$$\sin^2 2\theta_{23} = 1$$

$$\sin \theta_{13} = 0.1 \quad \theta = 0$$

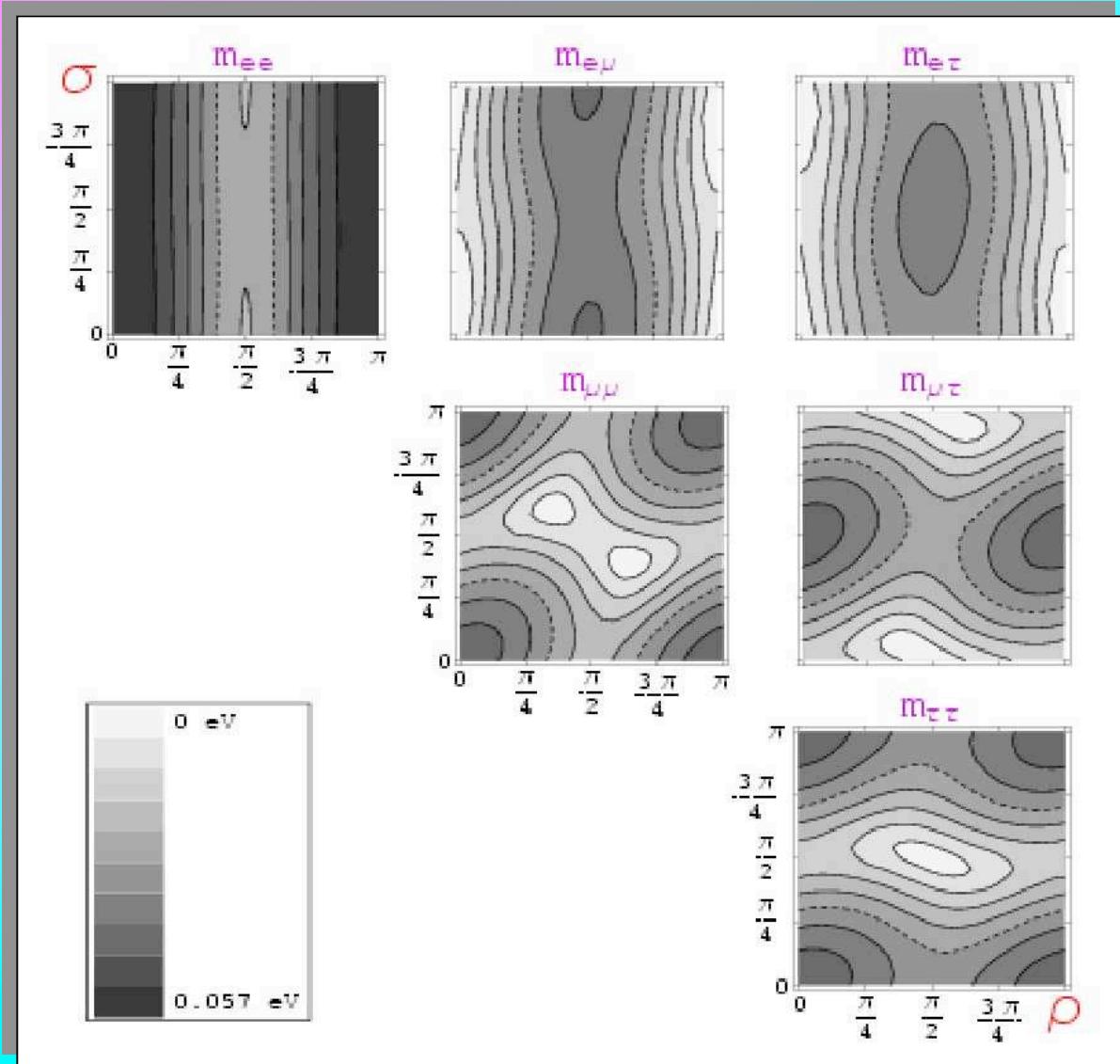
a). 
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

b). 
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

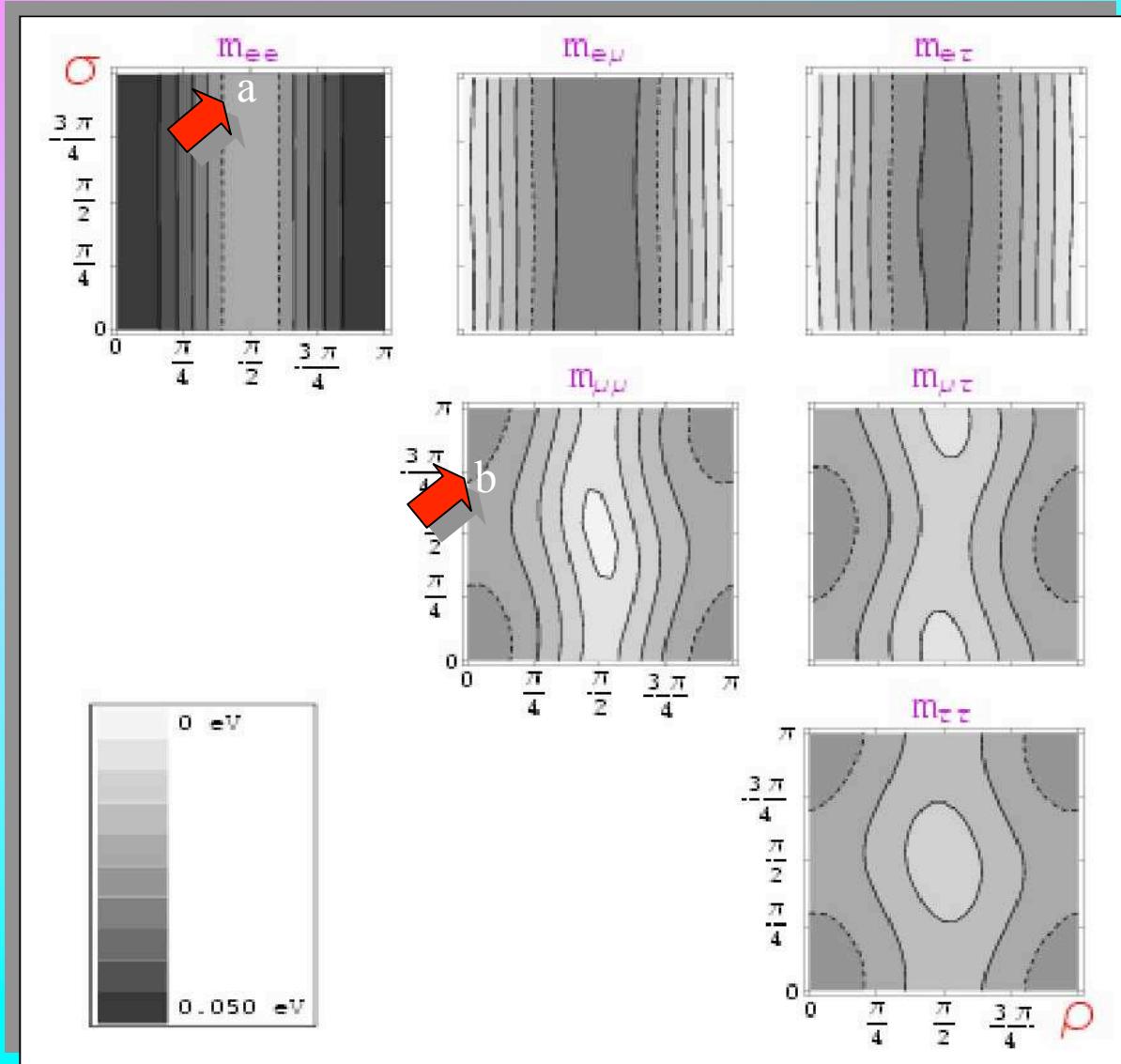
c). 
$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

# Inverted ordering

$m_3/m_2 = 0.5$   
 $m_3 = 0.029 \text{ eV}$   
 $\sin^2 2\theta_{23} = 1$   
 $\sin \theta_{13} = 0.1 \quad \theta = 0$



# Inverted hierarchy



$$m_3/m_2 = 0.1$$

$$m_3 = 0.005 \text{ eV}$$

$$\sin^2 2\theta_{23} = 1$$

$$\sin \theta_{13} = 0.1 \quad \theta = 0$$

a). 
$$\begin{pmatrix} 0.7 & 1 & 1 \\ 1 & 0.1 & 0.1 \\ 1 & 0.1 & 0.1 \end{pmatrix}$$

b). 
$$\begin{pmatrix} 1 & < 0.1 & < 0.1 \\ < 0.1 & 0.5 & 0.5 \\ < 0.1 & 0.5 & 0.5 \end{pmatrix}$$

# Observations

1). Large variety of different structures is still possible, depending strongly on unknown  $m_1$ , type of mass hierarchy, Majorana phases  $\theta_1$  and  $\theta_2$ , weaker dependence is on  $\sin\theta_{13}$  and  $\theta_{12}$ .

2). Generically the hierarchy of elements is not strong: within 1 order of magnitude.

Although, matrices with one or two zeros are possible.

3). Structures (in the flavor basis):

- with dominant diagonal elements ( $\sim I$ ), or dominant  $\begin{pmatrix} 1 & \\ & 0 \end{pmatrix}$ -blocks)
- with dominant e-row elements, (ee-,  $\begin{pmatrix} 1 & \\ & 0 \end{pmatrix}$ -,  $\begin{pmatrix} 0 & \\ & 1 \end{pmatrix}$ -) elements
- democratic structures,
- with flavor alignment,
- non-hierarchical structures with all elements of the same order
- with flavor disordering,

4). Typically hierarchical structures appear for  $\theta_1$  and  $\theta_2$  near 0,  $\pi/2$ ,  $\pi$

5). The structures can be parameterized in terms of power of small parameter  $\epsilon = 0.2 - 0.3$  consistent with Cabibbo mixing

L.Hall,  
H. Murayama,  
A.de Gouvea,  
F.Vissani,  
G. Altarelli,  
F. Feruglio,  
J.R. Espinosa

Anarchy?

# Neutrino mass and horizontal symmetry

Do neutrino results on masses and mixing or  
the neutrino mass matrix show some symmetry?

Is the neutrino mass matrix consistent with symmetries suggested for quarks?

$L_e - L_{\mu} - L_{\tau}$

Discrete  
symmetries  
 $A_4$   $S_3$   $Z_4$   $D_4$

$U(1)$

$SU(2)$

$SU(3)$

Treat quarks and  
leptons differently

in the Froggatt-Nielsen context  
can describe mass matrices both quarks  
and leptons.

$U(1)$  charges: discrete free parameters,  
also coefficients  $\sim O(1)$  in front of

Complicated higgs sector to break symmetry  
too restrictive...

## 4. How we might go ...

# Neutrality and mass

Minimalistic approach:

Relate features of the neutrino masses and mixing with already known difference of neutrino and quarks and charged leptons.

Minimal number of new concepts

Neutrality  
 $Q_{\square} = 0$   
 $Q_c = 0$

Right handed components, if exists, are singlet of  $SU(3) \times SU(2) \times U(1)$   
Unprotected by this symmetry

possibility to be a Majorana particle (Majorana mass term)

Can mix with singlets of the SM symmetry group

Can propagate in extra dimensions

Can have large Majorana masses  $M_R \gg V_{EW}$

$q - 1, SU(2)_L \times U(2)_R \times U(1)_{B-L}$

Properties of mass spectrum and mixing

Is this enough?

# Seesaw

T. Yanagida

M. Gell-Mann, P. Ramond, R. Slansky

S. L. Glashow

R.N. Mohapatra, G. Senjanovic

- $$\begin{pmatrix} 0 & m_D \\ m_D & M_R \end{pmatrix} \rightarrow$$

$$m_L = - m_D^T M_R^{-1} m_D$$

(type I)

$$m_D = Y v_{EW}$$

$$M_R = f_S \langle S \rangle = f_S v_R$$

- If the SU(2) triplet,  $L_L$ , exists with interaction  $f_L l^T l L_L + h.c.$ , then  $f_L l^T l L_L + h.c.$

$$m_L = m_L - m_D^T M_R^{-1} m_D$$

(type II)

- If  $L_L$  is heavy, induced VEV due to the interaction with doublet  $\langle L_L \rangle = \langle H \rangle^2 / M$
- In SO(10):  $L_L$  and  $S$  are in the same 126,  $f_L = f_S = f$

$$m_L = f_L \frac{v_{EW}^2}{v_R} - m_D^T f^{-1} m_D = \frac{v_{EW}^2}{v_R} (f_L - Y^T f^{-1} Y)$$

Flavor structure  
of two contributions  
correlates

# Variations on the theme

The number of the RH neutrinos can differ from 3

## Less than 3 ... 3x2-seesaw

(two RH neutrinos)

- one massless neutrino
- less number of parameters

## More than 3 ... Double seesaw

Three additional singlets S  
which couple with RH neutrinos

$$m_{\square} = - m_D^T M_D^{-1 T} \square M_D^{-1} m_D$$

Beyond SM: many heavy singlets  
...string theory

R.N. Mohapatra  
J. Valle


$$\begin{pmatrix} 0 & m_D \\ m_D & 0 \\ 0 & M \end{pmatrix} \begin{pmatrix} \square \\ \square^c \\ S \end{pmatrix}$$

- ≪  $M_D$  allows to lower the scales
- ≫  $M$
- ∼  $M_{GU}$ ,  $M \sim M_{Pl}$  explains intermediate scale

# Grand Unification and neutrino Mixing

- GUT provide large scale comparable to the scale of RH neutrino masses
- One can argue that GUT (+ seesaw) can naturally lead to large lepton mixing, or inversely, that large lepton mixing testifies for GUT

1. Suppose that all quarks and

i  
i  
(c)  
2. Suppose that all yukawa couplings

3. If Dirac masses are generated by  
a unique higgs multiplet (10 of  $SO(10)$ )

the up and  
down quarks  
have to have  
will be

As a result,

-  
-

4. In contrast to other fermions RH

1  
2  
3  
4  
5

5. Since those (Majorana type) Yukawa  
couplings are also of generic form they  
produce  $M$  with large mixing which  
leads then to large lepton mixing

Need to be  
slightly  
corrected

# Problem:

- Strong hierarchy of the quark and charged lepton masses

In this scenario  $m_D = \text{diag}(m_u, m_c, m_t)$

$$m_L = m_L - m_D^T M_R^{-1} m_D$$



Then for generic  $M_R$  the seesaw of the type I produces strongly hierarchical matrix with small mixing

# Possible solutions:



Type II seesaw:  
no dependence on  $m_D$



Special structure of  
 $M_R$  which compensate  
strong hierarchy in  $m_D$



Substantial difference  
of Dirac matrices of  
quarks and leptons  
 $m_D(q) \neq m_D(l)$

# Seesaw enhancement of mixing

Can the same mechanism (seesaw) which explains a smallness of neutrino mass also explain large lepton mixing?  
Large lepton mixing is an artifact of seesaw?

A.S.  
M. Tanimoto  
M.Bando,  
T.Kugo  
P. Ramond

Quark-lepton symmetry  
 $m_D \sim m_{\text{up}}$ ,  $m_L \sim m_d$ ,  
small mixing in Dirac sector

Special structure  
of  $M_R$



Large lepton mixing

## Two possibilities:



Strong ('`quadratic") hierarchy of the right handed neutrino masses:

$$M_{iR} \sim (m_{i \text{ up}})^2$$

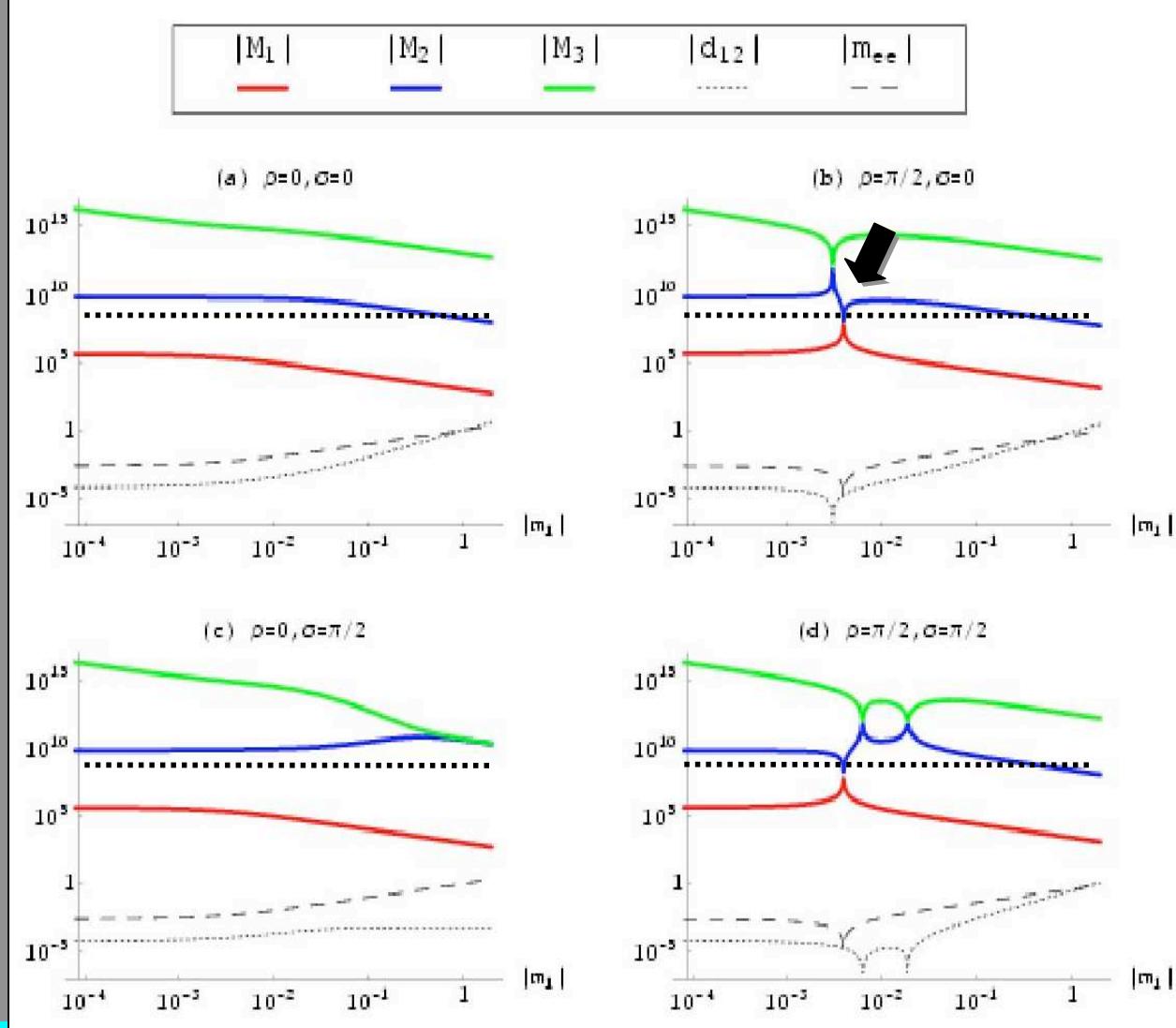


Strong interfamily connection (pseudo Dirac structures)

$$M_R = \begin{pmatrix} a & 0 & 0 \\ 0 & 0 & b \\ 0 & b & 0 \end{pmatrix}$$



# Masses of RH neutrinos



Leptogenesis  
gives strong  
restrictions

In the hierarchical  
case the lower  
bound on the  
lightest mass

$$M_1 > 4 \cdot 10^8 \text{ GeV}$$

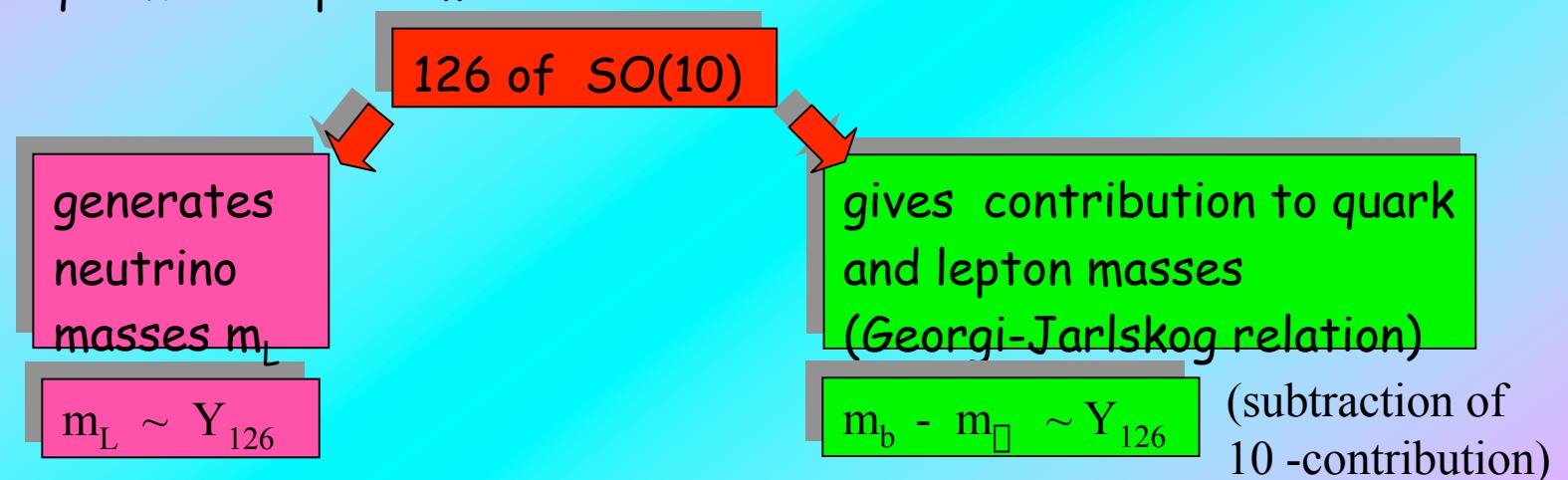
W. Buchmuller  
P. di Bari M. Plumacher,  
S. Davidson, A. Ibarra

Only in particular  
cases with strong  
degeneracy:  $M_1 = M_2$   
required asymmetry  
can be produced  
E. Kh. Akhmedov,  
M. Frigerio, A.S.

# Large mixing and type II Seesaw

- Structure of the mass matrix generated by the type II (triplet) seesaw can be related to quark and lepton masses

K. Babu, R. Mohapatra, Matsuda,  
B.Bajc, G. Senjanovic, F.Vissani  
R. Mohapatra, Goh, Ng



Large 2-3 mixing needs  $b - \bar{b}$  unification

$b - \bar{b}$  unification: element  $(Y_{126})_{33} \sim (Y_{126})_{23} \ll 1$   
--> large 2-3 lepton mixing

- Successful leptogenesis is possible with participation of the scalar triplet

T. Hambye, G. Senjanovic

# Single RH neutrino dominance

- Large mixing from the Dirac neutrino mass matrix

$$m_D = m \begin{pmatrix} * & * & \square \\ * & * & 1 \\ * & * & 1 \end{pmatrix} \quad M_R^{-1} = M \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(\* << □)

Seesaw gives:

$$m_\square = \begin{pmatrix} \square^2 & \square & \square \\ \square & 1 & 1 \\ \square & 1 & 1 \end{pmatrix}$$

S. F. King,  
R. Barbieri, Creminelli,  
A. Romanino,  
G. Altarelli, F. Feruglio,  
I. Masina

- In another version it may coincide with seesaw enhancement:  
Single RH neutrino dominance is realized when other RH neutrinos  
are heavy = strong hierarchy

# "Lopsided" Models

K. Babu, S.M. Barr,  
 C.H. Albright,  
 J.Sato, T. Yanagida,  
 N. Igres, S. Lavignac,  
 P.Ramond

- Large mixing follows from charged lepton mass matrix
- Non-symmetric mass matrices
- No contradiction with GUT:  
 in SU(5): LH components of leptons are unified  
 with RH components of quarks:  $5 = (d^c, d^c, d^c, l, \bar{e})$

$$m_D = m_u \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \bar{e} \\ 0 & -\bar{e} & 1 \end{pmatrix} \quad m_l = m_d \begin{pmatrix} 0 & \bar{e} & d' \\ \bar{e} & 0 & \bar{e} + \bar{e}' \\ \bar{e}' & -\bar{e} & 1 \end{pmatrix}$$

$\bar{e} \ll \bar{e}', \bar{e}' \ll \bar{e} \quad \bar{e} \sim 1$

Single lopsided

- Also possible in SO(10) if it is broken via SU(5)
- Double lopsided (for both large mixings) K. Babu, S. Barr
- Hybrid possibilities: large 2-3 mixing from charged lepton mass matrix  
 large 1-2 mixing from neutrino mass matrices

# Radiative enhancement of mixing

- Mixing is small at the Unification scale (similar to quark mixing) running to low energies  $\rightarrow$  enhancement of mixing.

Requirement:



e.g.

$$\frac{d \sin \theta_{23}}{dt} \sim (\sin \theta_{12} U_{11} D_{31} - \cos \theta_{12} U_{12} D_{32})$$

$$t = 1/8 \theta^2 \log q/M \quad D_{ij} = (m_i + m_j)/(m_i - m_j)$$

- Requires fine tuning of the initial mass splitting and radiative corrections
- In MSSM both 1-2 and 2-3 mixings can be enhanced. In SM ?
- If masses from Kahler potential: large mixing infrared fixed point
- Generation of small elements radiatively:  $|m_{12}|^2$ ,  $|\sin \theta_{13}|$

K. Babu,  
C.N. Leung  
J. Pantaleone  
P. Chankowski  
M. Pluciniak,  
J. Ellis, S. Lola,  
J. Casas,  
M. Lindner. ....

Enhancement  
when neutrinos  
become more  
degenerate

J.A.Casas,  
J.R. Espinosa  
I. Navarro

S. Petcov, A.S. A. Joshipura  
M. Lindner

# How to test Seesaw?

How to test existence of the heavy Majorana RH neutrinos?

## Leptogenesis

M. Fukugita, T. Yanagida

For hierarchical RH neutrino spectrum gives bound on

- Mass of the lightest RH neutrino  $\tilde{M}_{1R}$
- Effective parameter  $\tilde{m}_1$  which determines the washout effect  
Probe of  $(Y Y^+ )_{ij}$

W. Buchmuller  
P. di Bari  
M. Plumacher

$$M_{1R} > 4 \cdot 10^8 \text{ GeV}$$



$m_\square < 0.1 \text{ eV}$  excluding degenerate spectrum (?)

for type II seesaw: still possible

G.Senjanovic  
T. Hambye

## Renormalization effects of RH neutrinos

Renormalization effects between the scale  $M_{1R}$  and GUT  
e.g., on  $m_b - m_\square$  mass relation

F. Vissani, A.S.,  
H. Murayama, R. Rattazzi  
A. Brignole

# SUSY Seesaw

- Superpotential

$$W_{\text{lep}} = e^c{}^T Y_e l H_1 + \bar{\ell}^c{}^T Y \bar{l} H_2 + \frac{1}{2} \bar{\ell}^c{}^T M_R \ell^c$$

- 

Structures relevant  
for seesaw ( $Y, M_R$ )



structure of SUSY  
(slepton) sector

- Assumptions:
  - 1). Universal soft masses  
( $m_0^2, A_0$ ) at high scale  $M_X$
  - 2). No new particles apart from those in MSSM

- Contribution to the low energy left handed slepton mass matrices:

$$(m_S^2)_{ab} = m_a^2 \bar{\ell}_{ab} - \frac{1}{8\bar{\ell}^2} (3 m_0^2 + A_0^2) (Y^+)^{ai} (Y)_{ib} \log(M_X/M_{iR})$$

diagonal part

A.Masiero,  
F.Borzumatti  
L.J. Hall,  
V.A.Kostelecky,  
S. Raby  
F. Gabbiani,  
E Gabrielli,  
L. Silvestrini

# Testing SUSY seesaw

Rare decays

$$\square \rightarrow e \square$$

$$\square \rightarrow e \square$$

$$\square \rightarrow \square \square$$

A. Masiero  
F.Borzumati

Sneutrino-  
antisneutrino  
oscillations

Y. Grossman  
H.E. Haber

J.Ellis,  
S.Ferrara,  
DNanopoulos

.....

Electric  
dipole  
moments

# SUSY Seesaw

N. Arkani-Hamed  
H. Cheng, J. L. Feng,  
L.J. Hall

Sneutrino flavor  
oscillations

Slepton  
decays

A. Hinchliffe  
F.E. Paige

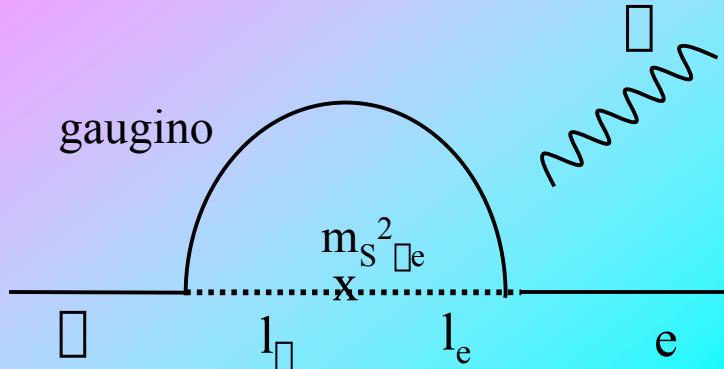
Reconstruction of

(Y<sup>+</sup> Y)

S. Davidson  
A. Ibarra

# Rare decays

- 



$$B(\text{gaugino} \rightarrow l \rightarrow \nu + e) = \frac{\alpha^3 |(m_S^2)_{l_e}|^2 \tan^2 \theta}{G_F^2 m_s^8}$$

$m_s = m_s(m_0, m_{1/2})$   
effective SUSY  
mass parameter

$$(m_S^2)_{l_e} = \frac{1}{8\alpha^2} (3 m_0^2 + A_0^2) (Y^+)_{l_i} (Y)_{ie} \log(M_X/M_{iR})$$

- If large lepton mixing originates from the Dirac matrix (lopsided models, versions of SRHN dominance)  
 $(Y)_{l_i}, (Y)_{ie}$  are large

$$B(\text{gaugino} \rightarrow l \rightarrow \nu + e) \sim 10^{-11} - 10^{-12}$$

At the level of present bound

A.Masiero,  
F.Borzumatti  
F. Gabbiani,  
E Gabrielli,  
L. Silvestrini

Beyond  
leading log:  
S. Petcov,  
S. Profumo,  
Y. Takanishi,  
C.E. Yaguna

# Other mechanisms

## Radiative mechanisms

Zee (one loop, generalized)  
Zee-Babu (two loops)  
Trilinear R- violating couplings

## Bi-linear R-parity violation

## Extra Dimensions

Large extra D (ADD)  
Warped extra D (RS)  
Infinite extra D (Dvali-Poratti)

## Dynamical symmetry breaking Technicolor

...

Can accommodate neutrino masses  
produce some interesting features

## Little Higgs

## Deconstruction

# Conclusions

Main open question:  
what is behind obtained results?  
Preference? Probably seesaw,  
and probably associated  
with Grand Unification.  
Although other mechanisms  
are not excluded and can  
give important or sub-leading  
*contributions*.

Enormous progress in determination  
of the neutrino masses and mixings,  
studies of properties of mass matrix.  
Still large freedom in possible  
structures exists which leads  
to very different interpretations.

How to check our ideas about neutrinos?  
Future experiments will perform precision  
measurements of neutrino parameters.  
Apart from that we will need results  
from non-neutrino experiments:  

- from astrophysics and cosmology
- from searches for proton decay  
and rare decays
- from future high energy colliders